BELLCOMM, INC.
955 L'ENFANT PLAZA NORTH, S.W. WASHINGTON, D. C. 20024

B69 09068

SUBJECT: A Study of the Passive Attitude Motion of Three Generic Types of Space Station Configuration Case 105-3

DATE: September 24, 1969

FROM: H. B. Bosch

NASW-417

ABSTRACT

Three models are considered for typical space station configurations: 1) a dumbbell shape, 2) a Y shape consisting of three half-dumbbells, and 3) a wheel shape consisting of a Y surrounded by a torus. Formulas are developed for calculating quotients of transverse to axial moments of inertia for the three types of configuration. The attitude motion of space stations spinning about a symmetry axis is studied, based on some results of P. Likins. It is concluded that such a space station can passively maintain a stable attitude (orientation) only if the spin axis is normal to the orbit plane. The influence of gyroscopes, a counterrotating hub, and aerodynamic or magnetic moments is not included.

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MEMORANDUM FOR FILE

I. INTRODUCTION

In the design and operation of an earth orbiting space station it may be physiologically and psychologically advantageous to the crew to subject them to "artificial gravity." This can be accomplished by creating a centrifugal acceleration at various distances from the hub of a spinning space station.

Generally, a space station will contain one or more sighting instruments (telescopes or other sensors) which need to be directed at the earth or at some other point in space. If the instrument is located such that its sensing axis is along the spin axis of the station, then it becomes important to examine the orientation of this spin axis relative to, say, an orbiting coordinate system. (Figure 1)

In order to gain some idea of the general behavior of the attitude motion of a spinning space station we will consider three representative configurations, namely: dumbbell, Y shape, and wheel. In each case the spin axis will move in space because of torques exerted on the spacecraft. These torques may be due to external forces (such as gravity differentials and aerodynamic or magnetic moments) or due to "fictitious forces" (such as Coriolis accelerations resulting from the orbital motion of the spacecraft). The effects of these torques on the attitude motion depends on the spin rate of the spacecraft and on the relationships between its principal moments of inertia. These relationships will be described next.

II. MOMENTS OF INERTIA

1. Dumbbell

The only assumption we will make for the model of a dumbbell is that its transverse moments of inertia be equal, or* $I_y^D = I_z^D$ (where the longitudinal axis is the x-axis). This model

*See List of Symbols.

could represent a space station which consists of two or more (not necessarily identical) modules which are rigidly connected along a common axis. The ratio of the axial to the transverse moments of inertia for the dumbbell will be designated

$$\varepsilon = I_X^D / I_Z^D \tag{1}$$

2. <u>Y Shape</u>

To construct a model for the Y shape we again start with a dumbbell as described in II.1 (Figure 2a), but with the additional assumption that the center of mass is at the geometric center. Let the symmetry axis of this dumbbell be the x-axis and, as above, let the ratio of its axial to transverse moments of inertia be ε. Let the inertia tensor for this figure be

$$\tilde{I}(0°) = \begin{pmatrix} I_X^D & 0 & 0 \\ 0 & I_Y^D & 0 \\ 0 & 0 & I_Z^D \end{pmatrix}$$

If this figure is rotated 60° about its mass center (Figure 2b), the inertia tensor becomes (Reference 1)

$$\tilde{I}(60°) = \begin{pmatrix} \frac{1}{4}\left(I_X^D + 3I_y^D\right) & \frac{\sqrt{3}}{4}\left(I_y^D - I_X^D\right) & 0 \\ \frac{-\sqrt{3}}{4}\left(I_y^D - I_X^D\right) & \frac{1}{4}\left(3I_X^D + I_y^D\right) & 0 \\ 0 & 0 & I_Z^D \end{pmatrix}$$

If it is rotated 120° (Figure 2c), the inertia tensor becomes

$$\tilde{I}(120°) = \begin{pmatrix} \frac{1}{4}\left(I_X^D + 3I_y^D\right) & \frac{-\sqrt{3}}{4}\left(I_y^D - I_X^D\right) & 0 \\ \frac{\sqrt{3}}{4}\left(I_y^D - I_X^D\right) & \frac{1}{4}\left(3I_X^D + I_y^D\right) & 0 \\ 0 & 0 & I_Z^D \end{pmatrix}$$

If these three dumbbells are superposed the resulting configuration would resemble six spokes 60° apart (Figure 2d). This can also be thought of as two identical Y shapes, superposed 180° apart. Since moments of inertia are additive, the inertia tensor for the Y shape (Figure 2e) is given by

$$\tilde{I}^{Y} = \frac{1}{2}\left[\ \tilde{I}(0°) + \tilde{I}(60°) + \tilde{I}(120°)\ \right]$$

Performing the indicated arithmetic results in

$$I_x^Y = \frac{3}{4}\left(I_X^D + I_Y^D\right)$$

$$I_y^Y = \frac{3}{4}\left(I_X^D + I_Y^D\right)$$

$$I_z^Y = \frac{3}{2}\ I_Z^D$$

Remembering that $I_y^D = I_z^D$ and

$$\varepsilon = I_x^D / I_z^D$$

the inertia quotient for the Y shape becomes

$$Q^Y = I_x^Y / I_z^Y = \frac{1}{2} + \frac{\varepsilon}{2} \qquad (2)$$

3. Wheel

To construct a model for the wheel configuration we start with a cylinder of length 2L and radius R, with an axisymmetric mass distribution (Figure 3a). Thus the density is given by

$$\delta = \delta(r) \qquad (3)$$

For such a cylinder the axial moment of inertia can be shown to be

$$I_x^C = 4\pi L \int_0^R \delta(r) r^3 dr$$

and the two (equal) transverse moments of inertia are

$$I_y^C = I_z^C = \frac{4}{3}\pi L^3 \int_o^R \delta(r) r dr + 2\pi L \int_o^R \delta(r) r^3 dr$$

The ratio, ε, of the axial to the transverse moments of inertia for the cylinder is then

$$\varepsilon = I_x^C / I_z^C = 6 / \left[2L^2 \left(\int \delta r dr / \int \delta r^3 dr \right) + 3 \right] \qquad (4)$$

Now we construct a Y shape out of three such cylinders, each of length L (Figure 3b). The moments of inertia for this configuration are*

$$I_x^Y = I_y^Y = \pi L^3 \int_o^R \delta(r) r dr + \frac{9}{2}\pi L \int_o^R \delta(r) r^3 dr$$

$$I_z^Y = 2\pi L^3 \int_o^R \delta(r) r dr + 3\pi L \int_o^R \delta(r) r^3 dr$$

so that, as before,

$$Q^Y = \frac{1}{2} + \frac{\varepsilon}{2}$$

where ε is given by (4).

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*These can be obtained by a method analogous to that used in II.2, or by direct evaluation of the three moment of inertia integrals involved.

Next we construct a torus out of a similar cylinder of length $2\pi L$ (Figure 3c). The moments of inertia for such a torus are

$$I_x^T = I_y^T = 2\pi^2 L^3 \int_o^R \delta(r) r dr + 5\pi^2 L \int_o^R \delta(r) r^3 dr$$

$$I_z^T = 4\pi^2 L^3 \int_o^R \delta(r) r dr + 6\pi^2 L \int_o^R \delta(r) r^3 dr$$

The inertia quotient is

$$Q^T = \frac{1}{2} + \frac{\varepsilon}{6} \tag{5}$$

where ε is given by (4).

Finally the wheel is configured by combining the torus with the Y shape (Figure 3d). Since moments of inertia are additive, we have for the wheel that

$$I_x^W = I_y^W = I_x^T + I_x^Y$$

$$I_z^W = I_z^T + I_z^Y$$

The corresponding inertia quotient for this wheel can then be derived as

$$Q^W = \frac{1}{2} + \frac{\varepsilon}{6}\left(\frac{3 + 2\pi}{1 + 2\pi}\right) \tag{6}$$

4. Some Comments

The models introduced above are not intended to correspond to any specific space station design. They are rather meant to be generic representations of three kinds of configuration for orbiting spacecraft. These idealized models are chosen in order to permit a relatively straightforward analytical treatment, and yet retain a certain degree of generality.

The inertia quotients, Q, calculated above are descriptive of the shape and mass distribution of the configurations; i.e., they describe static properties. There is, however, a dynamic difference between the dumbbell on the one hand and the Y shape and wheel on the other: if each of the configurations in Figures 2 and 3 is considered to be spinning about the z-axis, it becomes apparent that the wheel and Y shape are spinning about their symmetry axis, whereas the dumbbell is spinning about a transverse axis.

This difference is significant because the passive attitude motion of spacecraft *spinning about their symmetry axes* has been extensively studied. Notable among papers on this subject is the work of Likins (Reference 2). We shall use some of Likins' results to examine the attitude motion of the Y shape and wheel. On the other hand, the behavior of a symmetric object *spinning about an axis normal to its symmetry axis* has received very little attention. There are two papers (References 3 and 4) which attempt to address this problem. However, they fail to recognize (centrifugal and Coriolis) accelerations due to the orbital motion of the center of mass of the system, as well as the effects of gravity. Thus the spacecraft are represented as being stationary in a torque-free environment. Therefore, a future memorandum will treat the case of a dumbbell configuration which is rotating about a transverse axis. The attitude motion of a spinning Y and wheel will be examined in this memorandum.

The inertia quotients given by formulas (2), (5), and (6) are plotted on Figure 4 as a function of ε. This quotient, Q, is descriptive of the shape and mass distribution of the model, and of its orientation relative to a chosen coordinate system. The number ε, whether given by (1) or (4), can be thought of as describing the "slenderness" of the structure--in the sense that a small ε (approaching zero) corresponds to a very slender dumbbell or cylinder, and conversely.

III. ATTITUDE MOTIONS

1. Definitions

A convenient way to describe the *attitude* of a spacecraft which is spinning about a symmetry axis, is by the orientation of this spin axis relative to an orbiting coordinate system fixed to the spacecraft's center of mass (Figure 1). Assuming the center of mass to be moving in a circular orbit, A_1 points radially outward from the center of the earth, A_2 points in the direction of orbital motion, and A_3 is normal to the orbital plane so that (A_1, A_2, A_3) forms a righthanded, orthogonal triad. Thus the orientation of the spacecraft is indicated by the angles θ_1 and θ_2, where θ_1 is about A_1 (in the A_2-A_3 plane) and θ_2 is in the plane of the spin axis and A_1. The spin rate is indicated by $\dot{\theta}_3$.

Any attitude which the spacecraft can maintain in the absence of disturbances is an *equilibrium attitude*. If an equilibrium attitude is such that, under finite perturbations, the disturbed attitude remains close to the equilibrium attitude, then that equilibrium is called *Lyapunov stable*. If, no matter how small the perturbation, the disturbed attitude always moves far away from equilibrium, the equilibrium attitude is *unstable*. If the stability of an attitude has been determined by linearizing the equations of motion at the equilibrium point, it may still not be possible to determine analytically whether or not this stability is valid beyond the immediate vicinity of the point of linearization. In such a case the equilibrium is called *infinitesimally stable*.*

Figure 5 illustrates these concepts intuitively. A ball in the bottom of a bowl (Figure 5a) and a ball carefully balanced on the point of a pencil (Figure 5b) are both in equilibrium. The first, however, is Lyapunov stable whereas the second is unstable. If the tip of the pencil is blunted (Figure 5c) the ball could be considered to be infinitesimally stable.

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*The concept of infinitesimal stability does not necessarily describe the actual response of the attitude to a disturbance in general: the attitude still may or may not turn out to be Lyapunov stable in the above sense.

2. Equilibrium Attitudes and Their Stability

Likins (Reference 2) has studied the attitude motion, relative to an orbiting coordinate system, of a spacecraft spinning about a symmetry axis. Considering only inertial and gravitational torques, he has found that such spacecraft can spin in equilibrium in any of the following three types of attitude:

a. *cylindrical type* - so called because the spin axis is normal to the orbit plane and thus traces out a cylinder in space as the spacecraft goes through one orbit (Figure 6(a)). This attitude is described by

$$\left.\begin{aligned} \theta_1 &= 0,\ 180° \\ \theta_2 &= 0,\ 180° \end{aligned}\right\} \qquad (7)$$

b. *conical type* - where the spin axis is normal to the direction of motion but is inclined toward the earth, thus tracing out a cone (Figure 6(b)). The attitude and spin rate are now coupled, depending on the inertia quotient, as given by

$$\left.\begin{aligned} \theta_1 &= 0 \\ \cos\ \theta_2 &= -S/4(1-Q) \end{aligned}\right\} \qquad (8)$$

Here the spin parameter $S=\dot{\theta}_3/\Omega$ is the spin rate normalized to the orbital rate. It is numerically equal to the number of spacecraft rotations per orbit.

c. *hyperboloidal type* - where the spin axis is normal to the earth-spacecraft line but is inclined to one side, thus tracing out a hyperboloid (Figure 6(c)). The coupling between attitude, spin rate and inertia quotient is given by

$$\left.\begin{aligned} \cos\ \theta_1 &= -S/(1-Q) \\ \theta_2 &= 0 \end{aligned}\right\} \qquad (9)$$

where S is as before.

Likins has further derived relationships between spin rate and inertia quotients which determine the nature of the stability of each of the preceding equilibrium attitudes. With a minor modification* these results are shown in Figure 7.

IV. CONCLUSION

Assuming a rotation rate of at least one rpm (Reference 5), and considering that the shortest feasible period for an earth orbit is about 90 minutes, the spin parameter will take on values in excess of 90 and it will increase by at least 90 for each additional rpm. Figure 7 shows that, for $S>90$ (indeed even for $S>2$), no hyperboloidal or conical attitude (Figure 6) can be an equilibrium attitude. The cylindrical attitude, however, is guaranteed to be Lyapunov stable if the rotation is prograde (positive S), and at least infinitesimally stable if the rotation is retrograde (negative S).

Therefore, a space station rotating about its symmetry axis can passively maintain a stable attitude only if this axis is normal to the orbit plane.

The analyses reported here concern the passive attitude motion of a spinning spacecraft, subject to gravity and inertia torques. The inclusion of gyroscopic equipment (References 6, 7) onboard the spacecraft, as well as the influence of aerodynamic (References 8, 9) and magnetic torques (Reference 10), significantly complicates the analytical treatment but probably expands the stability regions indicated in Figure 7.

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Attachments
- References
- List of Symbols
- Figures 1 - 7

*Note that the inertia quotient Q is the reciprocal of Likins' parameter R. This linearizes the boundaries of the stability regions in his Figures 8, 10, and 11.

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LIST OF SYMBOLS

Symbols

$A_{1,2,3}$: orbiting coordinate frame

$\tilde{I}$: inertia tensor

$I_{x,y,z}$: moments of inertia about principal body axes x,y,z

L : length

Q : inertia quotient for models (equations 2, 5 and 6)

r,R : radial distances

S : normalized spin rate of spacecraft ($\dot{\theta}_3/\Omega$)

δ : mass density

ε : inertia quotient for dumbbell or cylinder (equations 1 and 4)

$\theta_{1,2,3}$: angles relating body and orbiting coordinate frames (Figure 1)

Ω : orbit angular speed

Superscripts

C : pertaining to cylinder

D : pertaining to dumbbell

T : pertaining to torus

W : pertaining to wheel

Y : pertaining to Y shape

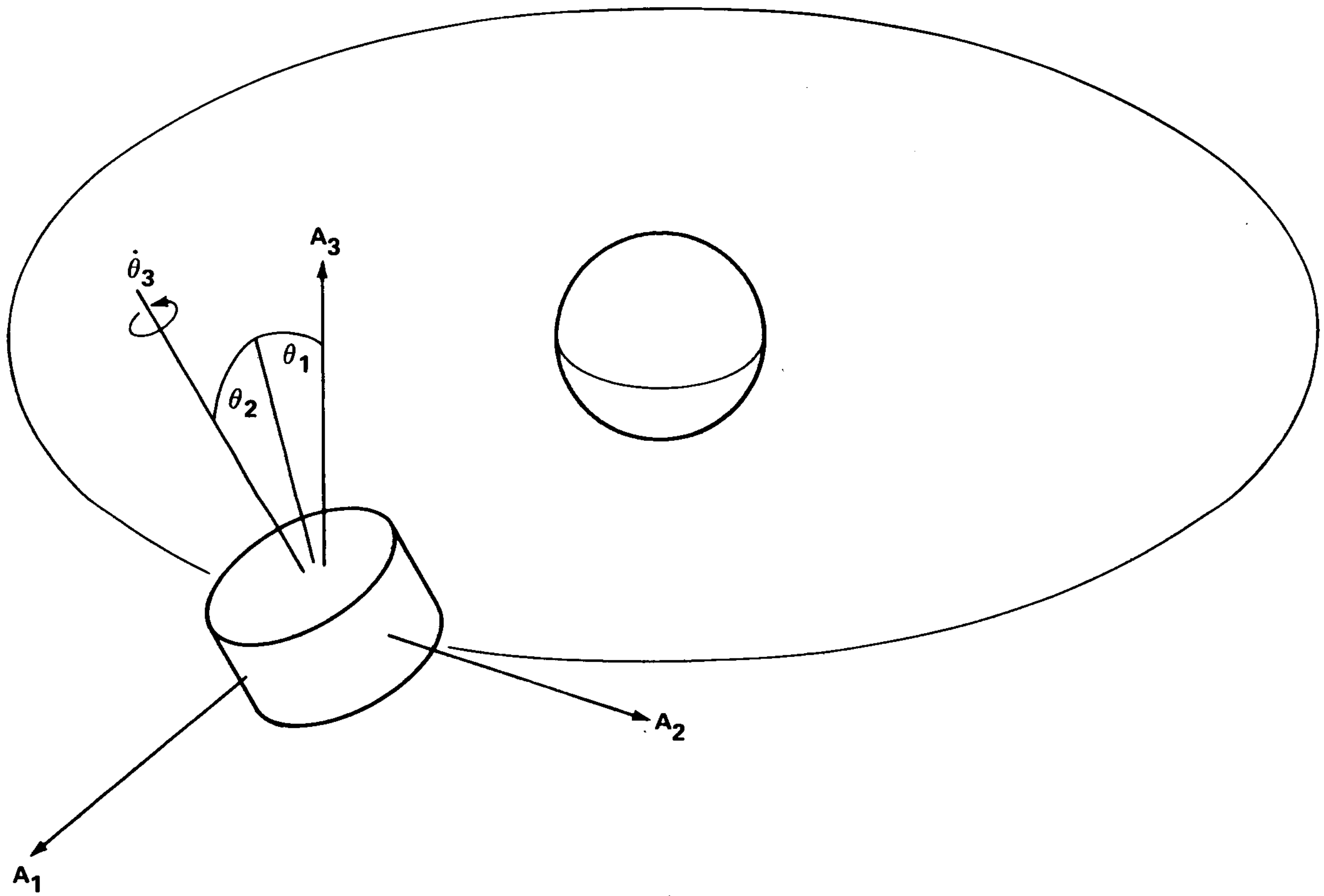


FIGURE 1 - ANGLES (θ_1, θ_2) DEFINING ORIENTATION OF SPIN AXIS RELATIVE TO ORBITING COORDINATE SYSTEM (A_1,A_2,A_3), AND SPIN RATE ($\dot{\theta}_3$).

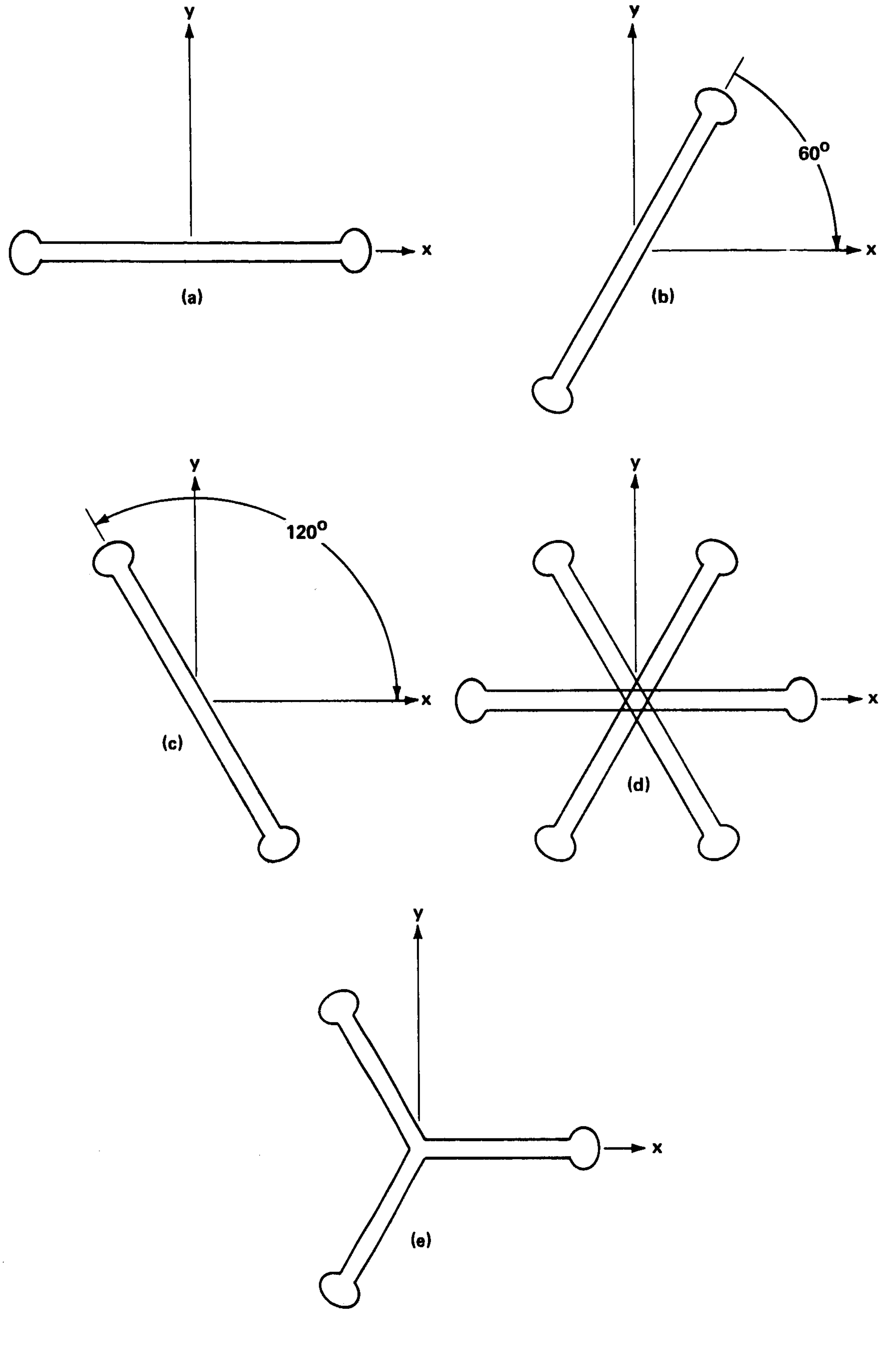


FIGURE 2 - GENESIS OF THE MODEL FOR THE Y-SHAPED CONFIGURATION (z-AXIS IS PERPENDICULAR TO PAGE, POINTING AT THE VIEWER).

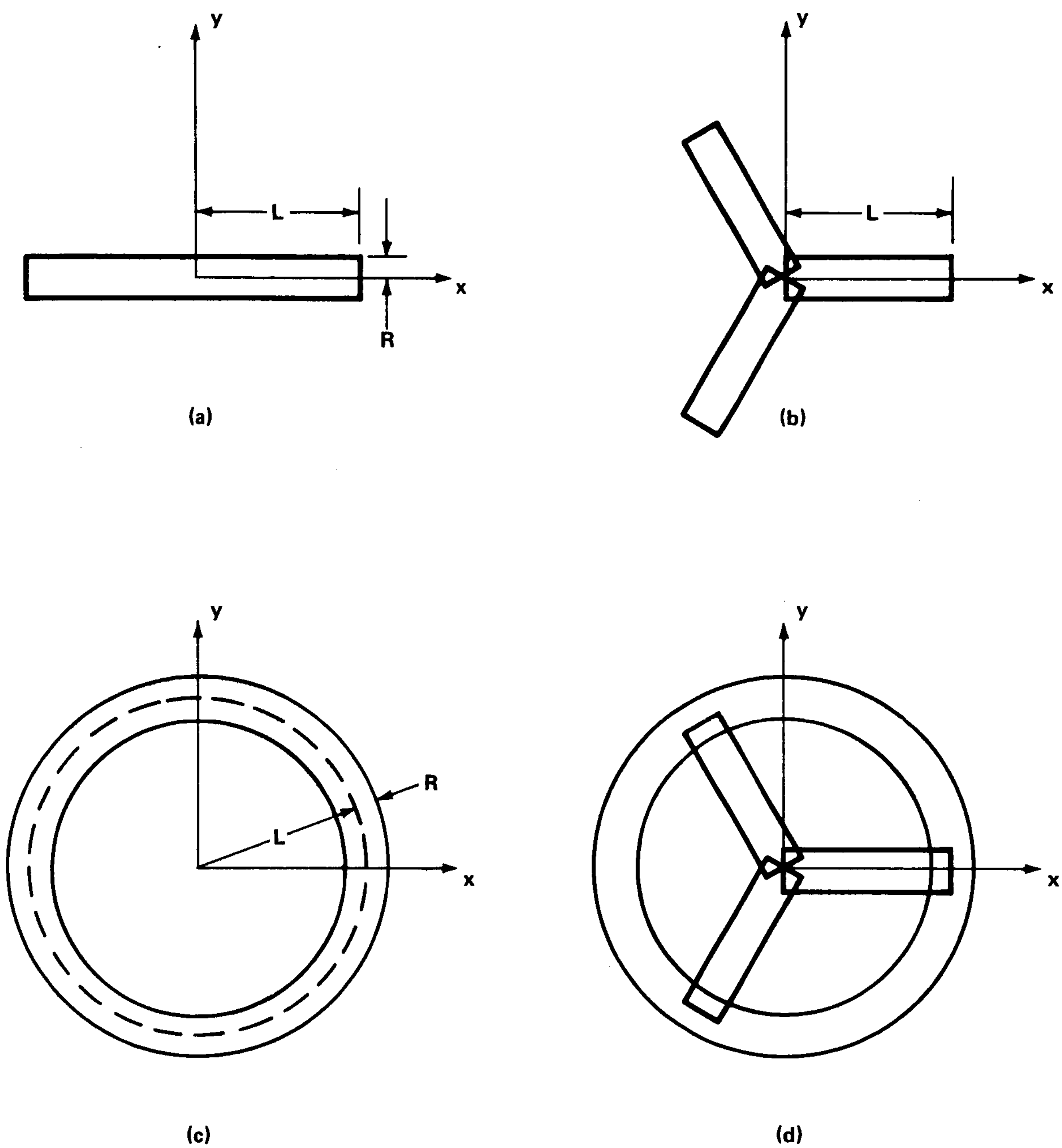


FIGURE 3 - GENESIS OF THE MODEL FOR THE WHEEL CONFIGURATION (z-AXIS PERPENDICULAR TO PAGE, POINTING AT THE VIEWER).

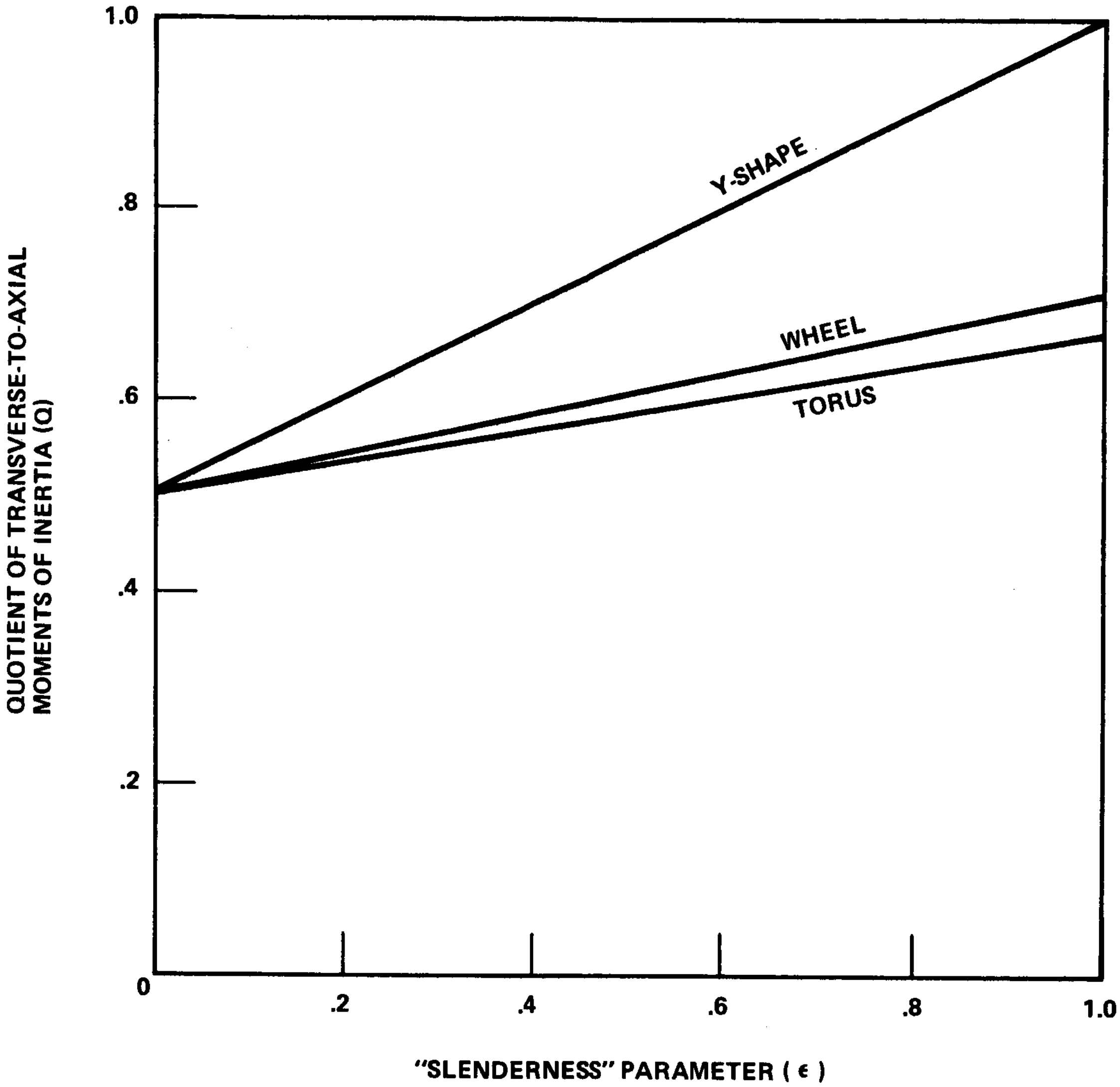
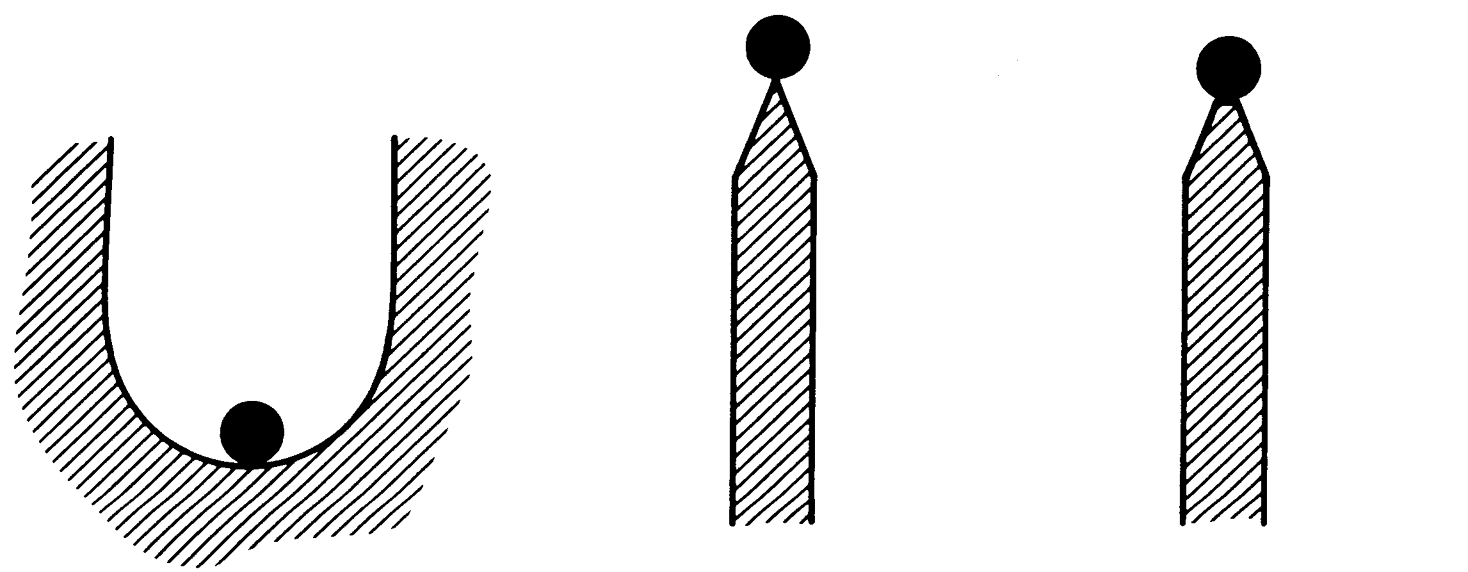
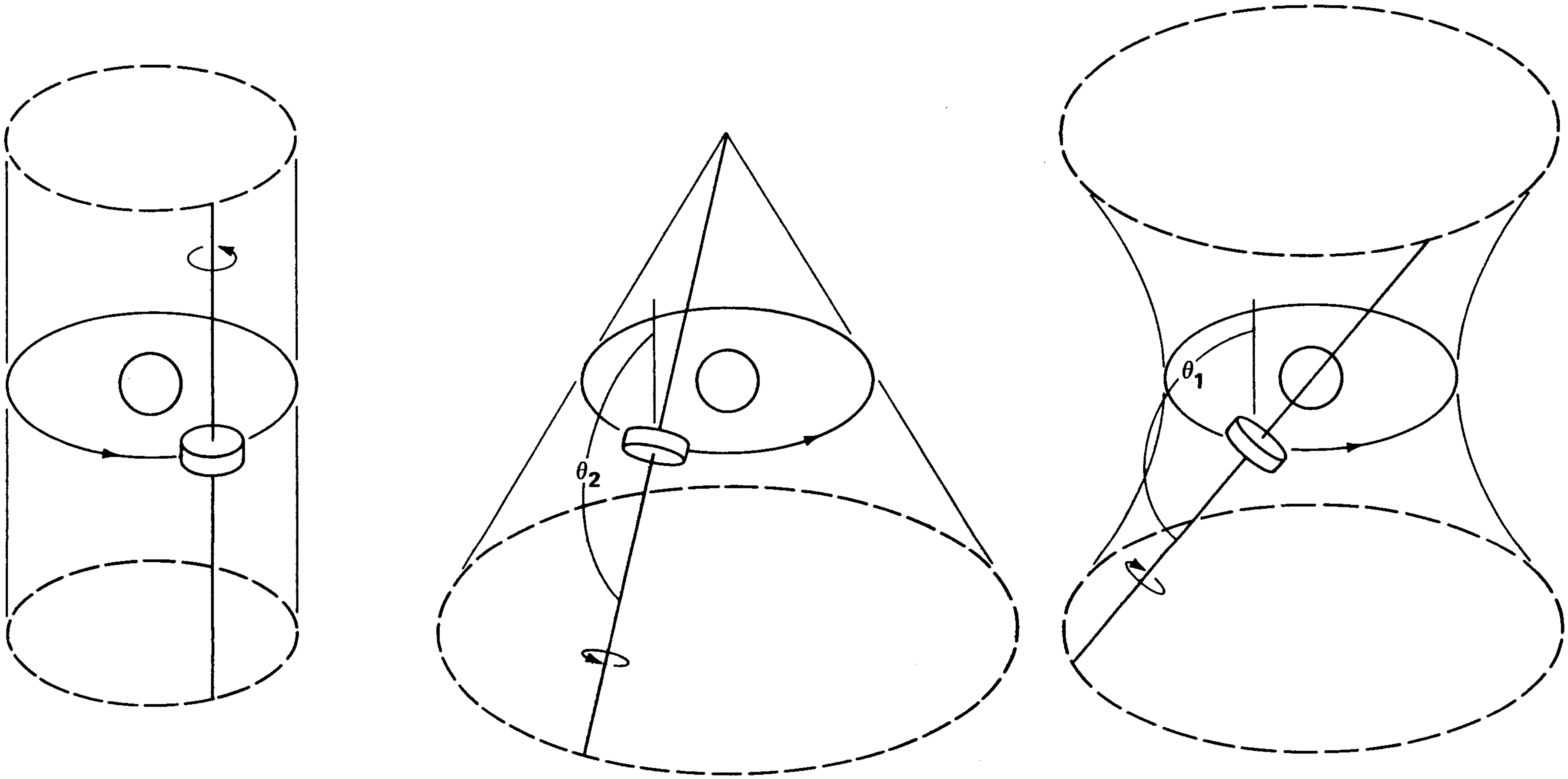


FIGURE 4 - INERTIA QUOTIENT AS A FUNCTION OF "SLENDERNESS" FOR THREE SPACECRAFT CONFIGURATIONS.



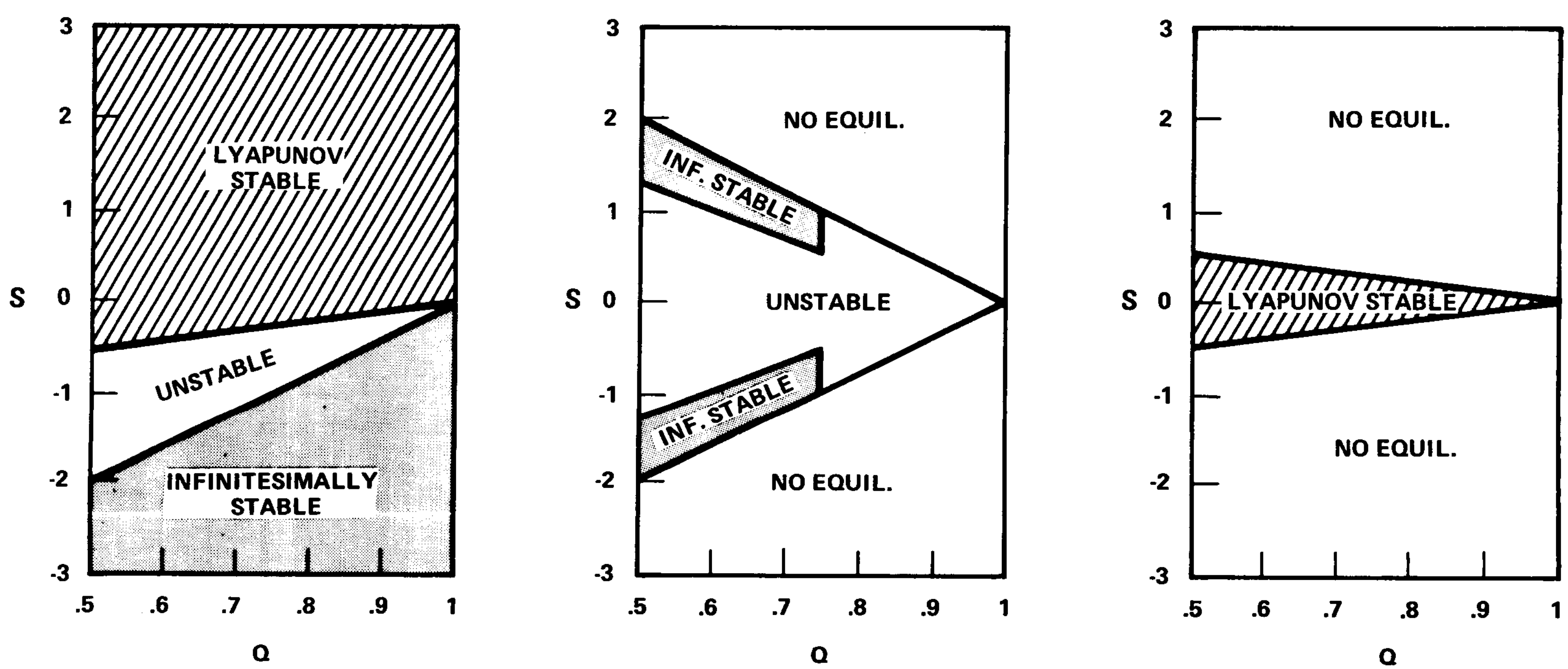
(a) LYAPUNOV STABLE (b) UNSTABLE (c) INFINITESIMALLY STABLE

FIGURE 5 - EXAMPLES OF THREE TYPES OF EQUILIBRIUM POSITION



(a) CYLINDRICAL (b) CONICAL (c) HYPERBOLOIDAL

FIGURE 6 - THREE TYPES OF EQUILIBRIUM ATTITUDE FOR SPINNING, AXISYMMETRIC SPACECRAFT



(a) CYLINDRICAL CASE (b) CONICAL CASE (c) HYPERBOLOIDAL CASE

FIGURE 7 - STABILITY DIAGRAM FOR THREE CASES OF EQUILIBRIUM ATTITUDE FOR A SPACECRAFT SPINNING ABOUT A SYMMETRY AXIS. (AFTER LIKINS)